Scientific Inquiry and Modeling
Linear Relationships

CONCEPT EXPLORATION

The sun rises and sets. You drop a glass of milk and it falls to the floor. Everywhere around you are patterns and cycles that can be modeled in order to attempt to accurately describe the behavior of the physical world.

A model is one tool that can be used to describe physical events and processes. However, scientific models can never be proven to be "true" and they can be rejected when experimental evidence demonstrates the inability of the model to make accurate predictions. Scientific models are constantly developing as new experimental evidence is collected.

Scientific models can be physical, conceptual, or mathematical. **Physical models** are objects that can be used to represent ideas about things that cannot be easily or directly observed. **Conceptual models** are ideas or systems of ideas that can be used to explain the behavior of variables. **Mathematical models** are equations that can also be used to make very specific predictions about the behavior of variables.

In this lesson, we will investigate the use of a linear equation as an example of a mathematical model.

Engagement Question

1. If you watch an hour of network television, how many minutes of commercials would you see?

The Challenge

You will develop a conceptual mathematical model that will allow you to predict the number of minutes of commercials that you would view based on the number of hours of television that you watch.

Your Ideas about the Challenge

2. Suppose you watched eight hours of television. How many total minutes of commercials would you see?

3. How did you use the answer to question 1 to answer question 2?

You could have created a simple mathematical model to answer question 2. This general equation could then be used to make predictions about the number of commercial minutes for any given number of television-viewing hours.

Number of commercial minutes	=	Number of commercial minutes per hour	•	Number of hours spent watching television

4. Write out a linear equation to model the relationship between the hours of television viewing and the resulting minutes of commercials. Use the numerical value that you selected to answer question 1 as your constant value for the number of minutes per hour. Be sure to include the units for your constant and indicate what each variable in the equation represents.

Check your work with your teacher.

CONCEPT DEVELOPMENT

In the exploration lesson you developed a simple mathematical expression that would allow you to predict the number of minutes of commercials that you would see based on the number of hours of television that you viewed. In this lesson you will develop a similar expression except that you will be required to do some work to determine the constant value to be used in the equation.

You will be working with a liquid substance that is very familiar to you. However, you might not be aware of the significance of water as the basis for the concept of density and the unit of heat, calories. Water is a very important substance that makes life, as we know it, possible.

Engagement Questions

1. What is the fundamental unit of mass in the metric system?

2. What volume of water would you need to have a kilogram of water?

3. Would the quantity of water that you answered question 2 with always have a mass of 1 kilogram. If not, why would it be different at times?

The Challenge

You will develop a mathematical model that will allow you to make predictions about the mass of water present in a container for a given volume.

Your Ideas about the Challenge

4. What is meant by the mass of an object or substance?

5. What is meant by the volume of an object or substance?

6. What mass do you think 10 ml of water would have? Explain how you made your guess.

The teacher will provide you with the following materials:
A graduated cylinder, a beaker, water, and an electronic balance

The Investigation

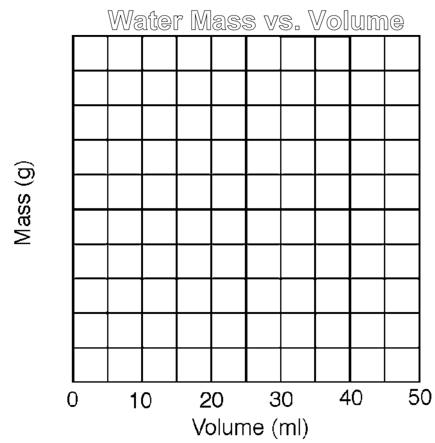
a. Determine the mass of an empty graduated cylinder using the electronic balance. Record this value in the data table seen below.
b. Place 10 ml of water in the graduated cylinder. Determine the mass of the cylinder with the water in it using the electronic balance.
c. Determine the mass of the water you placed in the cylinder. Record this value in the data table seen below.
d. Repeat steps “b” and “c” for volumes of 20 ml, 30 ml, 40 ml, and 50 ml of water.

Data

Empty Graduated Cylinder Mass (grams)	Mass of 10 ml of **water** (grams) mass of cylinder with water <u>-mass of empty cylinder</u> mass of just water	Mass of 20 ml of **water** (grams)	Mass of 30 ml of **water** (grams)	Mass of 40 ml of **water** (grams)	Mass of 50 ml of **water** (grams)

7. Do you notice a trend from one trial to the next? What do you think that trend is? Approximately how much does the mass appear to increase for each increase in volume?

8. Graph the mass of the water (y axis) vs. the volume of the water (x axis) on the grid provided below by plotting all four data points. You will need to label the y axis so that you get a dispersion of data and allow for the largest mass value that you received.



9. Draw a line that follows the "trend" of the data points to represent the behavior of the data.

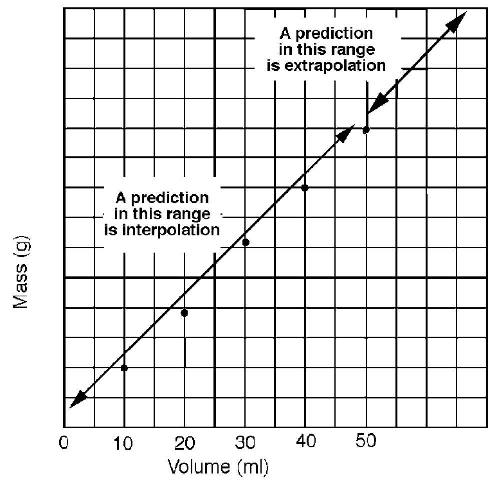
10. How would you describe the shape of the line. Is it roughly a straight line, a line curving upwards, a line curving downwards, or something else?

The graph of this data set is a valuable way to show the relationship between the mass for a given volume of water. In this case, as the volume of the water increased so did the mass. We could use this graph to make predictions.

For example, suppose you had 35 ml of water.

11. What would be the mass of 35 ml of water? How could you use the graph to make this prediction?

A scientist can make a prediction within a range of collected data, or outside of the range of collected data. Predictions within the range of collected data are called **interpolative** predictions. Predictions outside of the range of collected data are called **extrapolative** predictions. The figure that follows illustrates the difference between these kinds of predictions.



The shape of a best-fit line on a graph gives a scientist important clues about what the possible relationship is between variables. In this case, the best-fit line is pretty straight. A scientist would describe a relationship between mass and volume as **linear** because of this shape. Data that demonstrates this linear shape has some basic properties you will find very useful.

In the graph, the straight line had a certain steepness associated with it. This steepness is specifically referred to as **slope**. An observer can get a general idea about the slope by looking at the graph, but it is possible to determine a specific value for the slope for making precise predictions and comparisons.

If you are trying to determine a value for the slope of a straight line you could select two different points off of the line and then use the slope equation. It doesn't matter which two points you select since you should receive the same value for the slope as long as they come from the straight line. However, it is good technique to select two widely-separated points that fall on your resulting straight line and to be sure that you **don't** use data points.

The slope equation comes from the definition of slope as the amount of vertical rise with respect to the amount of horizontal run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

In order to use the slope equation you will need to select two points [(x_1, y_1) and (x_2, y_2)] from a straight portion of your resulting line on the graph that you did in questions 8 and 9.

12. Select two widely-separated points from the line that you drew on your graph. Write these in coordinate form below [(x_1, y_1) and (x_2, y_2)].

13. Use the slope equation ($\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$**) to determine the slope of your line. Be sure to carry through with the units that you used for both your x and y labels on your graph. Label you slope with appropriate units.**

A linear relationship can be **modeled** using a math equation. The slope-intercept form of a linear equation should be familiar to you:

$$y = mx + b$$

m is the symbol used to represent the slope of a line.
x is the symbol used to represent a given x-axis value.
y is the symbol used to represent a predicted y-axis value.
b is the symbol used to represent the y intercept, or place where the line crosses the y axis.

We will use this equation along with your average slope to predict a mass for a given volume of water.

Using a linear equation as a model , predict the mass for 45 ml of water.

Predicted Mass of Water (g)		Slope of Mass vs. Volume line (g/ml) from previous page		Given Volume of Water (ml)		y intercept from graph*
	=		•	45	+	

*probably = 0

14. What did you predict for the mass of 45 ml of water? Be sure to label your answer with the appropriate units.

The Investigation (continued)

a. Determine the mass of an empty graduated cylinder using the electronic balance. Record this value in the data table seen below.
b. Place 45 ml of water in the graduated cylinder. Determine the mass of the cylinder with the water in it using the electronic balance.
c. Determine the mass of the water you placed in the cylinder by subtracting the mass of the cylinder from the total mass of the cylinder and water together. Record this value in the data table that follows.

Data

Empty Graduated Cylinder Mass (grams)	Actual Mass of 45 ml of **water** (grams) mass of cylinder with water <u>- mass of empty cylinder</u> mass of just water

15. Did you receive exactly the value that you predicted using your mathematical model? If not, what do you think caused the difference?

In science classes, we will sometimes use a “percent error” or “percent difference” calculation to describe how “far off” the prediction was from the actual observed value. An equation that you can use to do this is shown below:

$$\%\ \text{error} = \frac{|\text{actual value} - \text{predicted value}|}{\text{predicted value}}$$

16. Calculate the percent difference between the actual observed mass for 45 ml of water and the predicted mass for 45 ml of water.

Check your work with your teacher.

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CONCEPT REFINEMENT

Review

You have now had two experiences modeling data with linear (y = mx +b) equations. This homework/worksheet will review and refine the ideas addressed in the previous activities.

In the first activity, you examined the relationship between mass and volume for a variety of water samples. You observed that there was a consistent trend between the mass and volume for water on a graph, and then calculated the slope (m) of the line on the graph. You used this slope as part of a linear equation model to predict a mass for any given volume of water

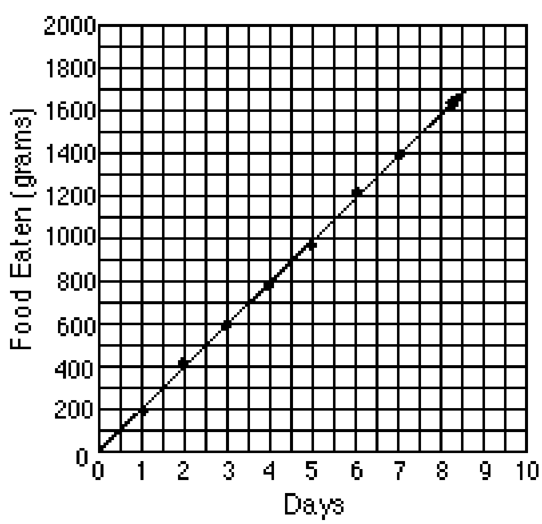
These investigations allowed you to use a linear equation to model data where the line was straight. The slope-intercept equation can be applied to many different types of linear data.

The following two sections will describe investigations conducted by students. It will be your job to evaluate their experimental design and describe the strengths and weaknesses of their efforts.

Sample Investigations

Takeisha plans to leave her dog with a friend while she takes a 10-day trip. She doesn't know how much food to leave with her friend, so she pays attention to how much food her dog eats for a week. The data she collected is illustrated below. She graphed the data on the grid below the data.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
200.g	215. g	180.g	190. g	205.g	220.g	175. g
Total Food Day I	Total Food by Day II	Total Food by Day III	Total Food by Day IV	Total Food by Day V	Total Food by Day VI	Total Food by Day VII
200.g	415. g	595. g	785. g	990. g	1,210 g	1,385 g



Takeisha observes the pattern in the data and says, "This is a linear relationship. I can predict how much food my dog will need for ten days using this line. I will divide 7 days by 1,385 grams to figure out the slope. I will then multiply the slope by 10 days. The answer should be a good prediction of the food my dog needs. "

Carefully read through the data, and Takeisha's plan.

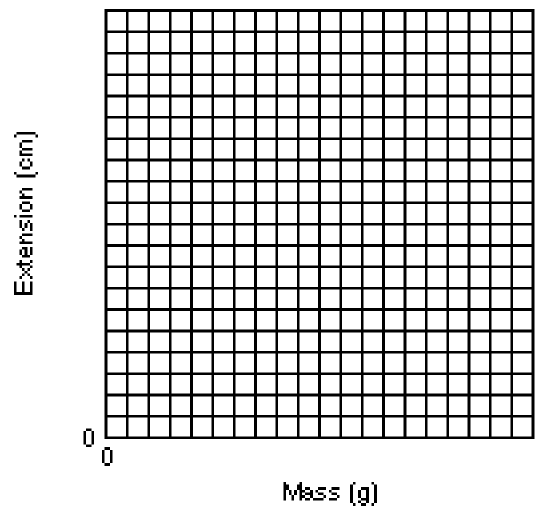
1. If you find any errors in Takeisha's plan, identify those errors and correct them so that an accurate amount of food is predicted.

2. Is this prediction an example of interpolation or extrapolation?

Check your work with your teacher.

Ben's science teacher passes out springs to all of the students and simply says, "Find out something about one of the springs." He sees some of the students measuring the mass of the spring, the length of the spring, and the diameter of the spring. He decides to find out how the spring will behave when mass is hung from the spring. The table below illustrates the data he collected.

Mass Hung (g)	Extension (cm)
100.	2.8
200.	5.4
300.	8.1
400.	10.8
500.	13.0



3. Graph the data on the provided grid.

4. Based on the shape of the line, what can you say about the relationship between the mass hung and the extension of the spring? Is this a linear relationship?

5. What mathematical equation could you use to model this data?

6. Calculate the slope of the line.

7. If you were to hang 250.grams from Ben's spring, what do you predict its extension to be?

Ben finds a second spring on the floor of his classroom. This second spring behaves in a similar way to his first spring (mass vs. extension is linear). The difference between this first spring and the second spring is that the slope of the mass vs. extension line for the second spring is 3.0 cm/g.

8. Does this mean that the second spring is easier or harder to stretch? Explain your answer. Feel free to sketch graphs or diagrams to aid in your explanation.

Check your work with your teacher.