Kinematics
Two Dimensional Vectors

CONCEPT EXPLORATION

A **vector quantity** is a quantity that has both magnitude (size) and direction. From the last lesson you determined that velocity has both magnitude (the speed) and direction. Therefore velocity is an example of a vector quantity.

Quantities that only have magnitude are considered to be **scalar quantities**. Mass is an example of a scalar quantity. There is no direction for the mass of an object.

There is a great deal of confusion regarding mass and its vector equivalent, weight. One reason for this is due to people who live in countries that use the metric system. They will typically express the weight of an object in grams or kilograms. Weight is a vector quantity which expresses the force of gravity acting on a body. Using mass units to express weight is incorrect. Having said that we are going to use mass units to express weight in this investigation since at this point you are unfamiliar with the metric units for weight.

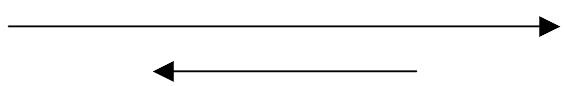
1. What is the direction for the weight of an object?

In this investigation you will explore how vector quantities can be added together. You will be introduced to the traditional method of drawing vectors graphically and you will see some of the more common orientations for two different vectors.

Engagement Question

2. You have already had the opportunity to investigate the difference between distance and displacement. Which of these quantities is a scalar quantity and which of these is a vector quantity? How do you know?

Vectors can be drawn graphically by using arrows. The magnitude of a vector is represented by the length of the arrow. The direction of the vector is indicated by the direction that the arrow points. For example; the vectors shown below represent two displacement vectors. One of these is an 8 meter displacement to the right and the other is a 4 meter displacement to the left.

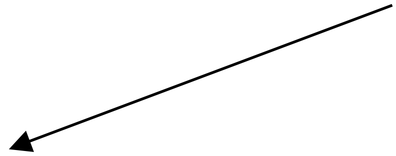


Whenever you draw arrows to represent vector quantities you should attempt to draw the vector quantities to scale. If there is a 1:2 (one to two) ratio between the magnitudes of two vector quantities then the length of the two vectors should also have a 1:2 ratio.

3. Draw two displacement vectors that are in the same direction. One of the vectors should represent a 20-meter displacement and the other should represent a 5-meter displacement.

You can move vectors around as long as you maintain their magnitude (length of arrow) and their direction. To maintain the direction of a vector when you move it, you should draw the new vector so that it is parallel to the original vector.

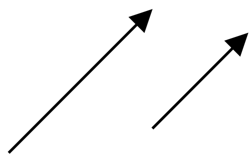
4. Move the vector you see below by redrawing it somewhere else in the space provided. Be sure to maintain both the magnitude and direction of the original vector.



Check your work with your teacher.

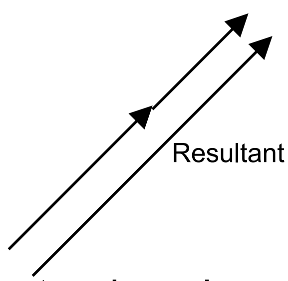
If you graphically add two vector quantities you can use the **head-to-tail** technique. As the name of this technique implies you would move the vectors so that the tail of one vector is at the head of the other vector. You always need to be sure to maintain the magnitude and direction of the original vectors.

5. The two vectors you see below are in the same direction. Graphically add these two vectors together by using the head-to-tail technique.



The sum of two vectors is called the resultant vector. The resultant vector always reaches from the tail of the first vector to the head of the second vector.

For example; the resultant for the two vectors that you added together using the head-to-tail technique would look like the diagram you see below.



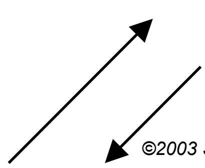
6. If the longer of the two original vectors shown above represented a 3 m displacement and the shorter of the two original vectors represented a 2 m displacement, what do you think is the magnitude of the resultant vector?

7. Evaluate the following student statement about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

"When you add two vectors together that are in exactly the same direction you just simply add them like you would add two numbers together."

Check your work with your teacher.

8. The two vectors shown below are in exactly the opposite direction. Use the head-to-tail technique to graphically add these two vectors together in the space provided. Be sure to show the resultant vector in your drawing.



9. If the longer of the two vectors shown above also represents a 3 m displacement and the shorter of the two vectors represents a 2 m displacement, what do you think is the magnitude of the resultant vector this time?

10. Draw a vector that is 3 cm long in the space below. Draw another vector that is 4 cm long and perpendicular to the first vector in the same space. Use the head-to-tail technique to add these two vectors together. Be sure to show the resultant vector in your drawing.

11. Measure the length of the resultant vector in cm and record this value below.
Data

Resultant Length (cm)

12. Can you think of a mathematical way to calculate the magnitude of the resultant of two vectors that are perpendicular to each other?

13. Evaluate the following student statements about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

<u>Student A</u>
"When you add two vectors together that are perpendicular to each other you can calculate the magnitude of the resultant by averaging together the lengths of the two original vectors."

<u>Student B</u>
"Since the resultant vector from the addition of two perpendicular vectors is the hypotenuse of the right triangle that forms, it should be bigger than either of the original vectors."

<u>Student C</u>
"You can calculate the resultant vector from two perpendicular vectors by using the Pythagorean theorem."

Check your work with your teacher.

14. What do you calculate if you use the Pythagorean theorem with the values of the original vectors given in the preceding exercise? Be sure to label your answer with the appropriate units.

$$(\text{Resultant})^2 = (\text{vector A})^2 + (\text{vector B})^2$$

$$R^2 = A^2 + B^2$$

$$R = \sqrt{A^2 + B^2}$$

$$R = \sqrt{(3\text{cm})^2 + (4\text{cm})^2} = \sqrt{9\text{cm}^2 + 16\text{cm}^2} = \sqrt{25\text{cm}^2} =$$

15. How does the solution that you received from the calculation above compare to the measured length of the resultant vector from the drawn 3 cm and 4 cm vectors? Is there a big difference between your two solutions?

Check your work with your teacher. ☐

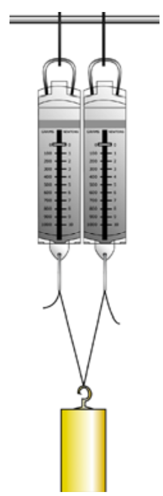
The Challenge

You will estimate the tension in a string when a mass is suspended from this string in a variety of ways.

Your Ideas about the Challenge

A 500-g mass is hung at the end of a string that is doubled over.

16. What do you think each of the spring scales will read in this situation?



At each lab station you will find the following:
Two 500-g spring scales, a 500-g mass, a length of string, and an apparatus for suspending your mass from the string.

The Investigation

a. Hang the 500 g mass in the middle of the string that is attached to the two spring scales.
b. Make sure that the spring scales are as close together as possible. Ideally the string would be straight down and doubled over.
c. Read the gram measure on each scale and record these below.

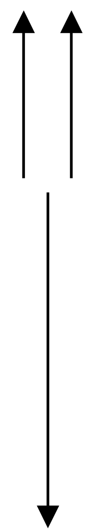
Data

Scale 1 (g)	Scale 2 (g)

17. Was there a big difference between the observed scale measurements on each of the scales? How can you explain this?

18. How did the sum of the two readings compare to the mass of the suspended weight?

To represent the investigation that you did graphically you would show the 500 g weight vector pointing straight down. The two tensions in the string would be directed upwards as shown to the right.



The arrangement for the suspended mass is said to be in equilibrium. **Equilibrium** in this case indicates that the system is at rest. In order to be at rest the sum of all of the vectors in this situation should be zero.

19. Explain in words or with a drawing how all three of these vectors would add up to zero. Hint: how does the combined length of the two "up" vectors compare to the length of the one "down" vector.

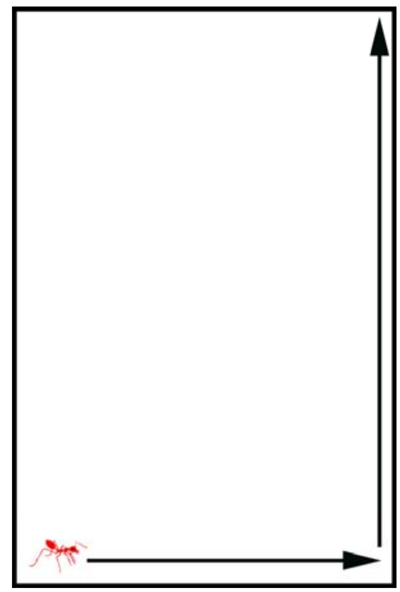
Check your work with your teacher.

Kinematics
Two Dimensional Vectors

CONCEPT DEVELOPMENT

From the exploration activity you saw how you could add vectors together that were in the same direction, the opposite direction, and perpendicular to each other. In this development activity you will investigate different mathematical techniques that can be used to find out more information about perpendicular vectors.

Engagement Question



Last night while you were sleeping an ant crawled along the edge of this piece of paper. The ant started at the bottom left-hand corner and crawled along the bottom to the other side. It then turned 90^{o} and crawled up the right side of this piece of paper to the upper right-hand corner.

1. What was the total displacement for this ant during this time. Explain what you did in order to arrive at a solution.

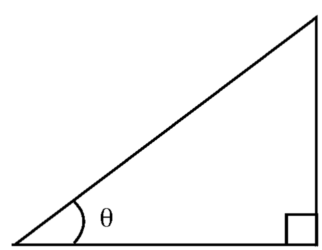
Check your work with your teacher.

The Challenge

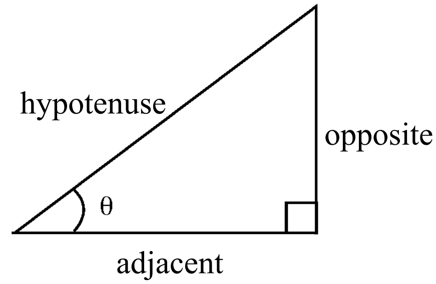
You will use trigonometric functions to solve problems that involve right triangles.

All trigonometric functions relate the lengths of two of the three sides of a triangle to one of the angles other than the right angle. In order to see which sides are to be used with each trigonometric function it is necessary to name each of the three sides.

In the diagram to the right you see a right triangle in standard position. The right angle (90^{o}) is labeled as well as the other angle to be used. The traditional way to label angles is with the θ (theta) symbol.



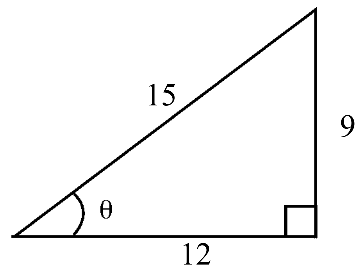
The longest side in the right triangle is the side across from the right angle. This side is called the **hypotenuse**. The side across from the labeled angle (θ is called the **opposite** side. Finally, the side that helps to make up the indicated angle, θ, along with the hypotenuse is called the **adjacent** side.



The **sine** (sin) function for an indicated angle, θ, is the ratio between the lengths of the opposite side in the triangle to the hypotenuse. This would be written as follows:

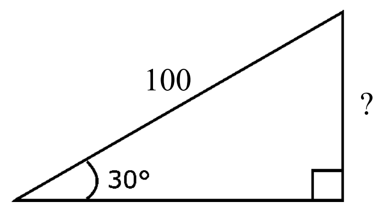
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Suppose the given right triangle had the length of sides shown.



Then $\sin\theta = \dfrac{9}{15} = 0.6$.

Suppose the right triangle shown to the right had the given hypotenuse and the given angle.



To determine the length of the opposite side you could use the sine function.

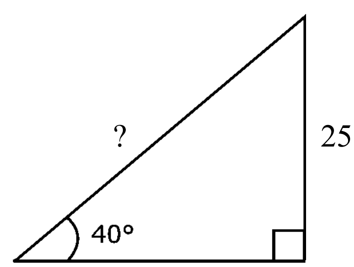
$$\sin 30^{\circ} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{?}{100}$$

$$? = (100)(\sin 30^{\circ}) = 50$$

You will need to use your calculator to determine the value of sin30°. You need to be sure to have your calculator in degree mode when you do this. Your teacher can help you to make sure that your calculator is in the proper mode.

2. What is the value of sin30^{0}?

The triangle that you see to the right has the given opposite side and the given angle.



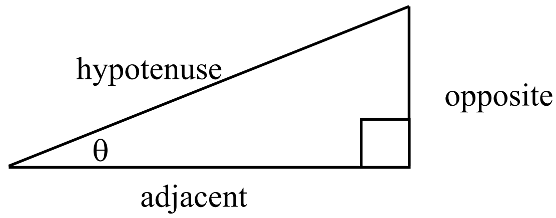
3. Use the sine function to calculate the length of the hypotenuse of the triangle that you see above.

Check your work with your teacher.

The other trig functions that you will need to know are the cosine function and the tangent function. The cosine function relates the ratio between the adjacent side and the hypotenuse with respect to a given angle in a right triangle. The tangent function relates the ratio between the opposite side and the adjacent side with respect to a given angle in a right triangle.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

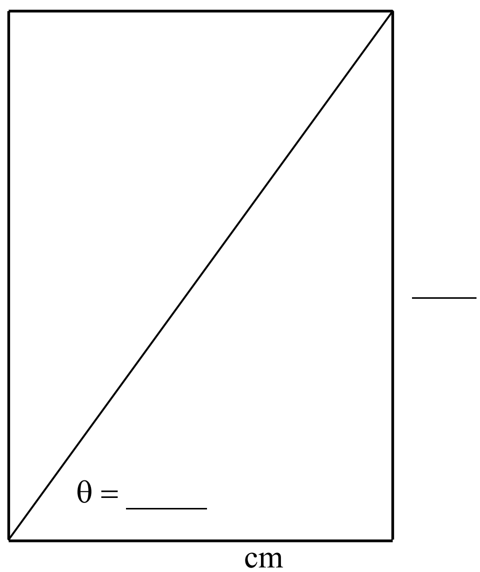
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



At each lab station you will find the following:
A metric ruler and a protractor

The Investigation

a. Measure to the nearest 10th of a centimeter the length and width of the piece of paper you are currently reading. Label the diagram you see below with the measurements.
b. Using a straight edge draw the diagonal line that goes from the bottom left-hand corner of this sheet of paper to the upper right-hand corner of this piece of paper
c. Use a protractor to measure the angle θ formed by the diagonal line you just drew and the bottom edge of the paper. Include this measurement in degrees in the diagram you see below.



4. With the measurements from your diagram use one of the three trigonometric functions to calculate the length of the diagonal line in your drawing. Be sure to show all of your calculations and use appropriate units.

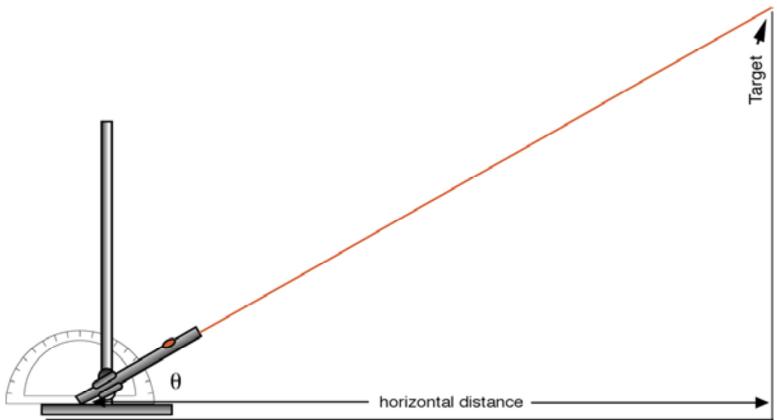
5. How did the solution that you received using the trigonometric function compare to the solution you received at the beginning of this investigation in the engagement question? Was there a big difference between your two solutions?

Check your work with your teacher. ☐

At each lab station you will find the following:
A laser pointer attached to a test tube clamp on an aluminum rod, a protractor, and a meter stick.

The Investigation (continued)

a. Find the target that your teacher has placed in a high position in your classroom.
b. Set up your laser pointer arrangement so that the bottom of the laser pointer touches the counter top and the laser strikes the center of the target when you switch it on.
c. Using the protractor, measure the angle that the laser pointer makes with respect to the counter top (horizontal). Record this measurement in the data section that follows.
d. Measure the horizontal distance from the point that the back of the laser pointer makes contact with the counter top to the point on the wall directly beneath the center of the target. Record this distance to the nearest 100th of a meter in the data section that follows.



Data

Measured angle (θ)

Horizontal distance (m)

6. Which side of the triangle (see the diagram) would you need to solve for in order to know how high the center of the target is above the counter top. Express the name of this side with respect to your measured angle θ.

7. Which trigonometric function relates the length of the side you mentioned in the preceding question to the side that you measured during the investigation (horizontal distance).

8. Use the trigonometric function you mentioned above to calculate the height of the center of the target above the counter top.

Check your work with your teacher.

Kinematics
Two Dimensional Vectors

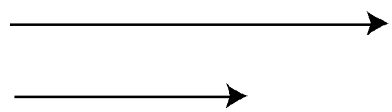
CONCEPT REFINEMENT

Review

You should know how to draw vectors to scale as well as how to correctly position them in order to add them together. When vectors are in the same direction you add them algebraically. When vectors are in the opposite direction you would subtract one from the other. When vectors are perpendicular to each other you can use the Pythagorean theorem to add them together.

You can use the trigonometric functions sine, cosine, and tangent in order to solve for various parts of right triangles.

The size of a vector quantity is indicated by the length of the vector arrow used to represent the given quantity. For example: if you are adding a 5 m displacement vector to a 3 m displacement vector you can draw the vectors so that one of them has a length of 5 cm while the other has a length of 3 cm.

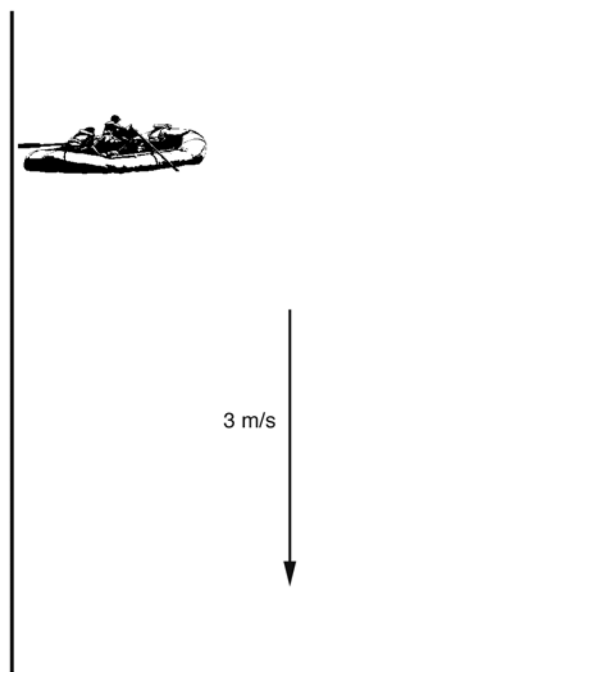


Shown to the right is an overhead view of a river that has a current of 3 m/s.

The magnitude and direction of the current is indicated by the vector shown at the bottom of the river diagram.

A raft in the river is capable of moving at 2 m/s when it is in still water. This raft is pointed in a direction that is directly perpendicular to the motion of the current.

The combined velocities of the raft and the river cause the raft to go in a direction other than directly across this river.



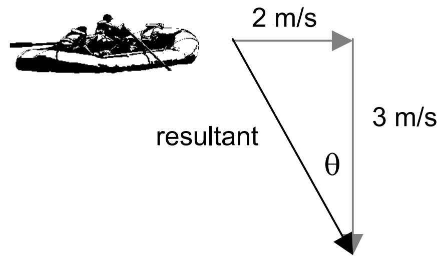
1. In the diagram above draw the velocity vector that would represent the 2 m/s velocity of the raft when it is in still water.

2. Using the head-to-tail technique rearrange the two vectors in the same diagram so that you can correctly add these vectors together. Be sure to include the resultant vector in your diagram.

3. Use the Pythagorean theorem to solve for the actual velocity of the raft due to the combined motions of both the raft and the river.

Check your work with your teacher.

One way you could have arranged the vectors in the preceding activity is shown to the right. An angle, θ, is included in this diagram that represents the resultant direction for this raft with respect to the current of the river.



The tangent function relates the 3 m/s current vector to the 2 m/s raft vector with respect to the angle θ shown in the diagram above.

$$\tan\theta \; = \; \frac{\text{opposite}}{\text{adjacent}} \; = \; \frac{2\frac{m}{s}}{3\frac{m}{s}} \; = \; \frac{2}{3}$$

What happened to the units shown in the equation above?

To solve for the angle θ you will have to use an inverse trigonometric function. This inverse function, sometimes referred to as arctangent (arctan), is written $\tan^{-1}$.

$$\tan^{-1}(\tan\theta) \; = \; \tan^{-1}\left(\frac{2}{3}\right)$$

Inverse tangent of tangent just leaves you with the angle θ on the left side of the equation ($\tan^{-1}(\tan\theta) = \theta$

$$\theta \; = \; \tan^{-1}\left(\frac{2}{3}\right)$$

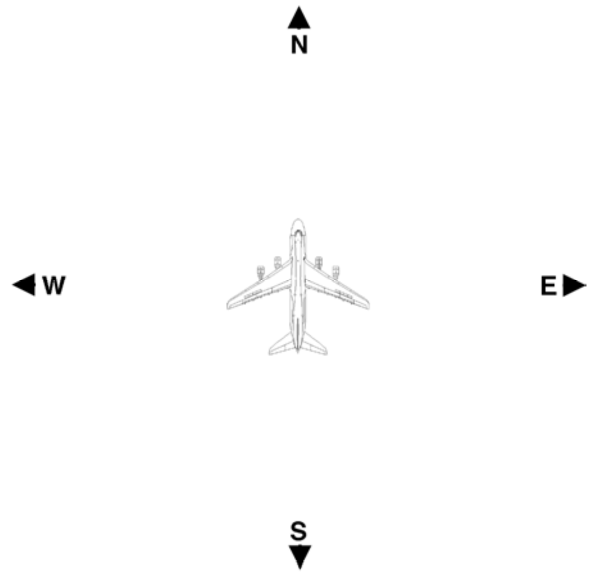
To calculate the value of the angle θ using your calculator, you should first make sure that your calculator is in degree mode. You can access inverse functions on your calculator by using the 2nd function or inverse key. Your teacher can assist you in finding this button.

Calculate the measure of the angle θ in degrees.

Check your work with your teacher.

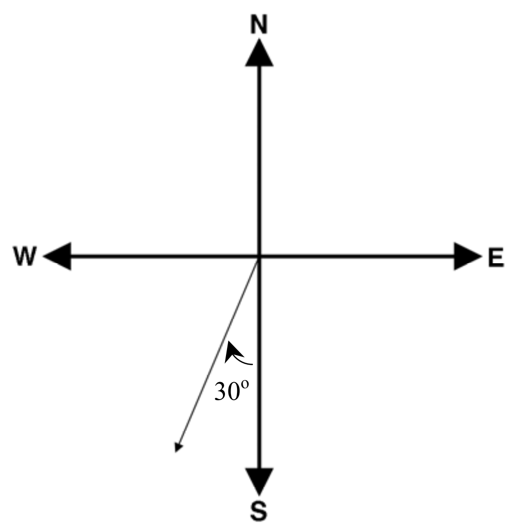
A jet aircraft is moving at 500 miles/hour directly north. It encounters a cross wind that is blowing towards the west at 100 miles/hour. On the diagram provided to the right draw the velocity vectors that represent the velocity of the aircraft and the velocity of the wind.

Move the two velocity vectors so that they are head to tail. Draw the resultant vector in your diagram as well.

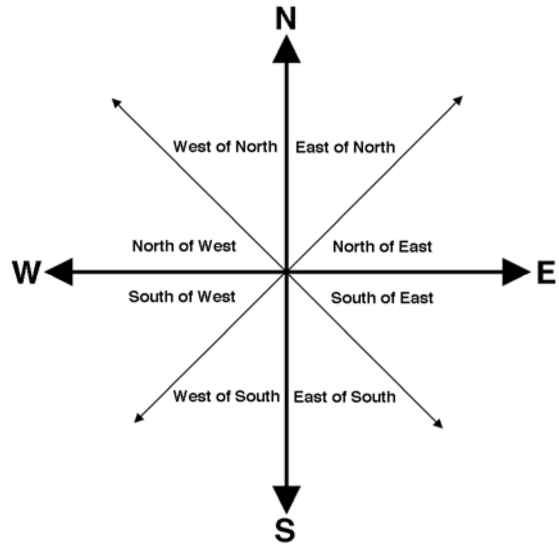


Calculate the magnitude for the resultant velocity. Show all of your calculations and include the appropriate units.

To express the direction of a vector with respect to the North-South-East-West direction axes you should express the number of degrees that the vector is away from one of the axes. For example the following angle could be expressed as being 30° West of South.



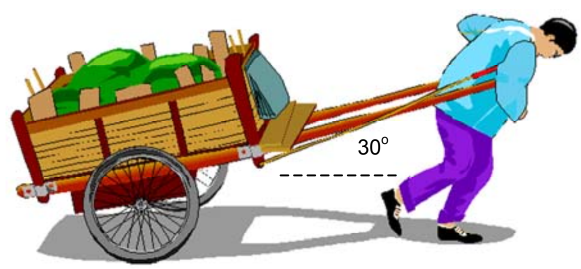
You can use the graphic provided to the right to help you to understand which direction expression to use.



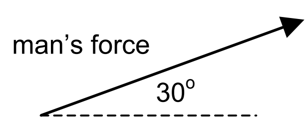
Calculate the direction of the resultant velocity of the jet. Express this answer in degrees and with respect to the direction axes.

Check your work with your teacher.

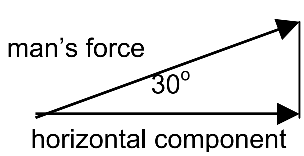
A man pulls on the yoke of a wagon at an angle of 30^o with respect to horizontal.



The man exerts a force on the wagon as indicated by the vector that you see to the right.



This vector can be broken down into what is known as horizontal and vertical components. A vector component represents the amount of a vector that acts in a particular direction. The directions that we are usually interested in are the vertical direction and the horizontal direction. In order to determine how much of the man's effort goes into propelling the wagon forward you would look at the amount of his effort that is in the direction that the wagon is being moved. This is the horizontal direction.



The trigonometric function that relates the two vectors shown in the preceding diagram is the cosine function.

$$\cos 30^0 = \frac{\text{adjacent}}{\text{hypotenuse}}$$

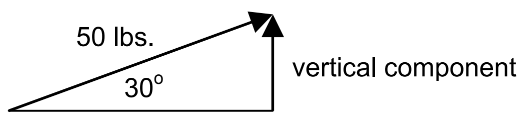
In order to solve for the magnitude of the horizontal component, that represents the effort of the man that moves the wagon forward, you would multiply both sides of this cosine expression by the hypotenuse.

$$\text{adjacent} = (\text{hypotenuse})(\cos 30^{o})$$

If the man was pulling with a force of 50 lbs. Then the amount of this force that is in the direction that the wagon is being moved can be calculated as you see to the right.

$$(\text{hypotenuse})(\cos 30^{0}) = (50\text{ lbs.})(\cos 30^{0}) = 43.3\text{ lbs.}$$

Since the man is pulling on the wagon in a direction that is 30^{o} with respect to the horizontal he is also pulling upwards on the wagon. The amount of his effort that lifts upwards on the wagon is represented by the vertical component of his force. This is illustrated by the vertical vector that you see included in the diagram below.

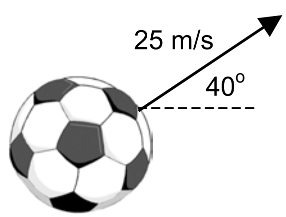


Which trigonometric function relates the two vectors that you see above with respect to the given 30^{o} angle?

Using the trigonometric function that you mentioned in the preceding question calculate the magnitude of the vertical component of the man's effort.

Check your work with your teacher.

A ball is kicked so that it leaves the ground at an angle of 40^{o} with respect to horizontal and at a velocity of 25 m/s.



Calculate the horizontal velocity component for this soccer ball at the moment that it leaves the ground.

Calculate the vertical velocity component for this soccer ball at the moment that it leaves the ground.

Check your work with your teacher.