Energy
Energy Conservation
Applications 2

CONCEPT EXPLORATION

In this lesson you will be continuing to explore the concept of conservation of energy but with another form of energy. The form of energy that we will explore in this unit will be that of elastic potential energy, specifically the energy that can be stored in a spring.

Engagement Question

A student compresses a spring against a table top.

1. Does this spring have energy as the student holds it in this compressed position? Explain how you know the answer to this question.

The Challenge

You will determine the relationship between the energy available in a compressed spring and the amount of compression of the spring.

Your Ideas about the Challenge

A ball bearing is launched by a spring. The ball bearing is launched a second time with the spring compressed twice as much as before.

2. Do you think that the ball bearing will move faster after the second launch? How much faster will the ball bearing roll the second time (same speed, twice as fast, three times as fast, four times as fast . . .)?

At each lab station you will find a computer with a motion detector, a low-friction cart, and a spring attached to the top of the rear of the cart over the top of a metric ruler.

Your teacher will make sure that there is a velocity-time graph on your computer display. The graph should display negative velocity.

The Investigation

a. Set up the motion detector so that it is approximately 1.5 meters away from your cart.
b. Start the motion detector program.
c. Insert a pencil into the forward loop of the spring. While holding the rear of the lab cart in place, stretch the spring to the 2-cm mark on the metric ruler. Release the rear of the cart so that the cart is propelled forward directly towards the motion detector.
d. Stop the cart before it reaches the motion detector.
f. Repeat this procedure 3 more times. Record the maximum velocities attained by the cart (drop the negative sign) in the data table that follows.
g. Repeat steps “b” through “f” stretching the spring to the 4-cm mark.

Data Tables

2-cm stretch

Launch #	Maximum velocity attained (m/s)
1	
2	
3	
4	

4-cm stretch

Launch #	Maximum velocity attained (m/s)
1	
2	
3	
4	

3. Calculate the average maximum velocity that the cart attained with the 2-cm stretch.

4. Calculate the average maximum velocity that the cart attained with the 4-cm stretch.

5. Roughly how much bigger was the 4-cm velocity compared to the 2-cm velocity (twice as big, three times as big, four times as big, . . .)?

6. Evaluate the following student statement about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

"The cart almost went twice as fast when we launched it with the doubled stretch. If there wasn't any energy lost due to air resistance and the heating up of the spring then it would have gone exactly twice as fast."

Check your work with your teacher

7. What kind of energy did the cart have after it was launched by the spring?

8. Write the equation for the kind of energy that you wrote for #7 above.

9. If the velocity is doubled for an object, by how much does its kinetic energy increase?

10. If energy is transferred from the stretched spring to the launched cart, by how much must the energy stored in the spring have increased when the spring was stretched twice as much?

11. If the amount of stretch in the spring is tripled, by how much do you think the stored energy in the spring would be increased?

12. Evaluate the following student statement about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

"When we doubled the stretch in the spring we doubled the velocity of the cart. Since velocity is squared in the kinetic energy equation, the amount of stretch of a spring must also be squared in the spring potential energy equation."

Check your work with your teacher ☐

The symbol that stands for the amount of expansion in a spring is the letter "x". This quantity is squared in the spring potential energy equation. The amount of spring potential energy is directly proportional to the amount of expansion (or compression) of a spring.

$$PE_{spring} \propto x^2$$

To turn this proportion into an equation there must be a "constant" of proportionality. This constant depends on the size of the spring that is used.

13. Would a large spring give more or less energy to a projectile than a small spring? Explain your answer.

The spring constant (k) for a large spring is bigger than the spring constant for a small spring. The amount of spring potential energy is also directly proportional to the spring constant.

$$PE_{spring} \propto kx^2$$

We can determine the spring constant for a spring by using Hooke's law F = kx (k = F/x). This will be part of the investigation in the development lesson that follows.

Check your work with your teacher ☐

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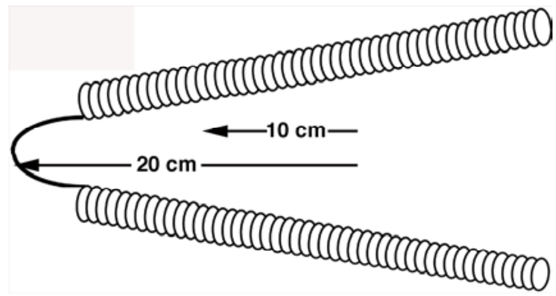
CONCEPT DEVELOPMENT

During the exploration activity you determined that a stretched spring has stored energy. You should also know that a compressed spring has the same kind of stored energy. These springs have potential energy because they have the ability to do work on an object. This potential energy can be changed into other forms of energy.
In this development exercise you will calculate how much kinetic energy a projectile acquires from a spring slingshot. You will then determine the velocity given to the projectile so that you can calculate the range of the projectile.

Engagement Question

A spring slingshot is first stretched 10 cm and then 20 cm.

1. How much more spring potential energy does this slingshot have when it is stretched 20 cm, compared to the amount that it has when it is stretched 10 cm?



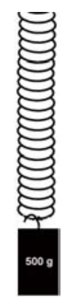
The Challenge

You will determine the speed that a ball bearing will acquire when it is fired from a spring slingshot.

Your Ideas about the Challenge

A 500 g mass is hung from the bottom of a spring so that the spring stretches due to the weight of the mass.

2. If you hang a 1-kg mass from the end of the spring do you think that the spring will stretch twice as much? Explain your answer.



Your teacher will suspend various size masses from the bottom of a spring slingshot. The distance that the springs are stretched by the increasing mass will be shared with the class.

Fill out the table that you see below.

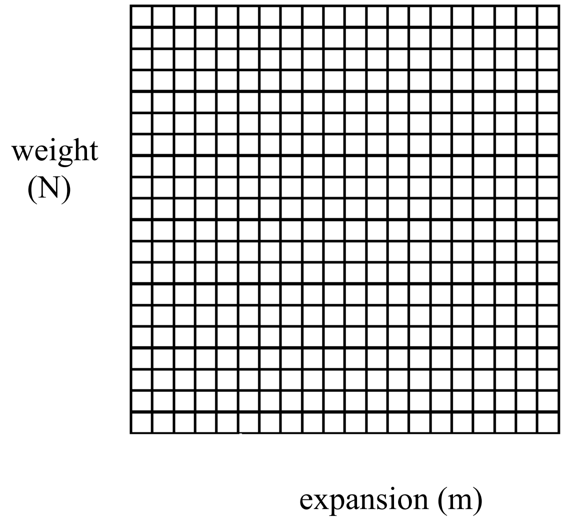
Suspended mass (kg)	Weight of suspended mass (N)	Spring expansion (cm)	Spring expansion (m)

3. Graph the weight of the suspended mass (N) with respect to the spring expansion expressed in meters.

You will need to appropriately label both the vertical and horizontal scales so that all of your data points will fit on the graph but spread out as much as possible.

Be sure to use “regular” intervals on the two axes.

Your teacher can help assist you in setting up your graph with the data that is made available to you.



4. Does the relationship between the weight of the suspended mass and the expansion of the spring appear to be linear?

5. Using a straight edge, draw the best-fit line that passes through the trend of your points.

To find your spring constant (k) you will use the equation for Hooke’s law: F = kx. You can do this by calculating the slope of the resulting best-fit line on your weight vs. expansion graph (F vs. x graph).

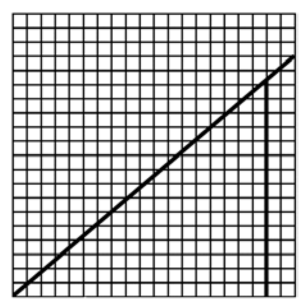
6. Calculate the slope of your weight vs. expansion graph. Show the coordinates for the two points that you selected to use and be sure to carry through with all of your units.

$$k = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} =$$

The mathematical expression that you can use to calculate the amount of spring potential energy that is stored in a spring can be derived from the graph that you just drew. Since we are dealing with a variable force you can’t just simply multiply the force times the displacement of the spring in order to determine how much work is being done on the spring. The area between the force-displacement line and the x-axis is a graphical representation of the amount of work done on the spring.

If you draw a vertical line from the force-displacement line on you graph that intersects the x-axis you get a familiar geometrical shape.

7. What is the geometrical shape that you see bordered by the force-displacement line, the vertical line, and the x-axis seen on the grid to the right?



8. What is the area equation for the shape that you see on the grid to the right?

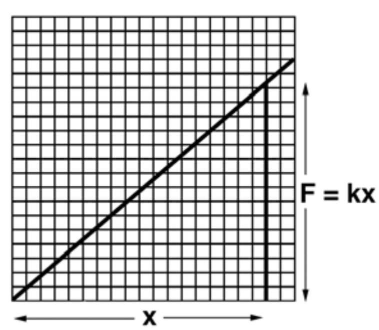
The “height”, of the shape shown to the right, can be represented by the product, kx, from Hooke’s law. The base of the shape is the displacement of the spring (x). If you use these variables to calculate the area of this shape you would get the following expression:

$$\text{area} = \tfrac{1}{2}\text{bh} = \tfrac{1}{2}(\text{x})(\text{kx}) = \tfrac{1}{2}\text{kx}^2$$

Therefore the work done on a spring when it is stretched by a force (kx) is:

$$\text{work} = \tfrac{1}{2}\text{kx}^2$$

This is also the mathematical expression for the potential energy stored in either a stretched or compressed spring.

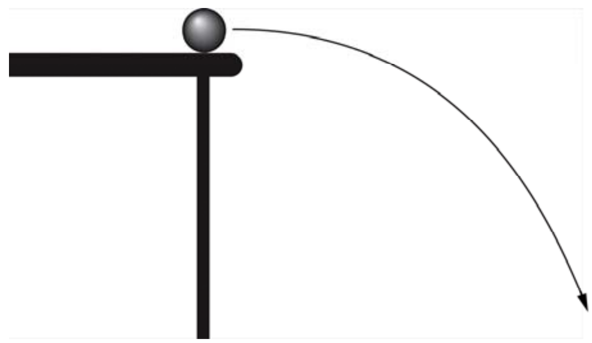


9. Calculate how much spring potential energy is contained in the spring slingshot if it is pulled back 20 cm (0.2 m). Use the equation $PE_{spring} = 1/2kx^2$. Your spring constant "k" that you should use is the answer you got to question 6. Be sure to carry through with all of your units and label your answer with the appropriate units.

10. How much kinetic energy would a projectile acquire from this spring slingshot if it were pulled back 20 cm and fired in the horizontal direction? Explain how you know this.

11. Calculate the speed of your projectile using the equation KE = $1/2mv^2$ ($v = \sqrt{\frac{2KE}{m}}$). Your teacher will tell you the mass of the projectile that will be used, expressed in kilograms.

If your projectile were fired off of a table in your classroom, the time it was in the air would be determined by the height of the tabletop above the floor.



12. Measure the height of the tabletop that your teacher has selected in the classroom. Record this height in meters (m) in the data table below.

Height of tabletop (m)

13. Calculate how long the projectile will be in the air. Use the equation $t = \sqrt{\frac{2d}{a}}$. Be sure to use appropriate units and to label your answer with appropriate units.

14. Calculate the "range" of your ball bearing by using the equation $d = v_0t$. The velocity will be the velocity that you calculated in question 11. The time that you should use will be the time that you calculated in question 13. Be sure to carry through with all of your units and to label your answer with the appropriate units.

Check your work with your teacher

Your teacher will demonstrate the range of the projectile by firing it from the tabletop using the spring slingshot.

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CONCEPT REFINEMENT

Review

You should know about three kinds of energy. You should be able to calculate the amount of each of these kinds of energies that an object might have depending on the kind of information that is available to you.
The table shown below will help to summarize the three kinds of energy along with the equations for their expressions.

Energy	Definition	Equation
Kinetic	**The energy of motion**	**$KE = 1/2mv^2$**
Gravitational Potential	**The energy of vertical position**	**$PE = mgh$**
Spring Potential	**The energy stored in a compressed or expanded spring**	**$PE_{spring} = 1/2kx^2$**

Any combination of these three forms of energy can be converted into another combination of these three forms of energy. If energy is conserved, then you can use the concept of conservation of energy to solve problems that involve the conversion of energy.

A 5 kg bowling ball, rolling at 3 m/s, strikes a compressible spring that has a spring constant of 400 N/m.

1. What kind of energy does this system have before the bowling ball strikes the spring? How do you know this?

2. What kind of energy does this system have at the moment that the bowling ball has fully compressed this spring and halts momentarily before it reverses direction? Explain how you know.

If energy is conserved then the total energy of this system before the bowling ball strikes the spring is equal to the total energy that this system has at the moment that the bowling ball has fully compressed the spring.

total energy before = total energy after

3. Substitute the mathematical equations for the kinds of energy that this system has before and after into the basic conservation of energy equation shown above.

4. Simplify the equation you wrote for question 3.

5. Using algebra, solve the equation that you wrote for the preceding question, for the symbol that stands for the compression of the spring.

6. Plug in the values, that were given at the beginning of this problem, along with their units into the equation that you wrote above. Calculate the value for the maximum compression of the spring and label your answer with the appropriate units.

Check your work with your teacher.

A 500 g (0.5 kg) baseball is compressed a vertical distance of 20 cm (0.2 m) against a vertical spring.

The spring has a spring constant of 800 N/m.

7. What kind of energy does this system have before the baseball is released?

The baseball is released so that it shoots straight up into the air.

8. What kind of energy does the baseball have at its highest point?

9. Use the concept of conservation of energy to determine how high the baseball will rise after it is released. Be sure to carry through all of your units and label your answer with the appropriate units.

Check your work with your teacher.