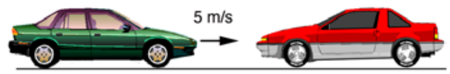
Impulse and Momentum
Elastic Collisions

CONCEPT EXPLORATION

When two objects collide together there is a large range of possible outcomes that can occur. Factors such as the sizes of the two objects, the composition of the two objects, their speeds before contact, and the angle at which they strike one another all play important roles in determining the outcome of such an interaction.
Have you ever collided with another person? If so, there are things that you could have done to minimize the damage.

Engagement Question

An unoccupied car, with the engine not running and left in neutral (not in gear), rolls through the parking lot at a speed of 5 m/s. This car collides head on with another car of equal mass that was parked on level ground with no parking brake on and had also been left in neutral.



1. What do you think will happen to each car after the collision?

The Challenge

You will investigate how equal-mass objects behave when they collide together in a one-dimensional bouncing collision.

Your Ideas about the Challenge

2. When can an object exert a greater force on another object, when it is moving fast or slow? Explain your answer.

At each lab station you will find the following materials:
2 lab carts with bumpers

The Investigation

a. Place one of the lab carts on the table top approximately 50 cm away from the other lab cart.
b. Both carts should be placed so that their bumpers face one another.
c. Make sure that there is at least 1 meter of counter space behind the target cart.
d. Push on one of the lab carts so that it moves directly towards the other cart so that their bumpers will strike one another.
e. Stop the target cart after it has traveled a short distance.
f. Repeat this process several times using different collision speeds. **Don't push the carts so hard that they will sustain damage as a result.**

3. Describe what happened to each cart after the collision. Be sure to take note of the relative speed of the moving cart before the collision, compared to the speed of the target cart after the collision. Was the initial speed of the moving cart considerably bigger or was it about the same as the final speed of the target cart?

4. What was the momentum of each cart before the collision? This would be the time after you had pushed on one cart but before the collision took place. Assume that each cart has mass, m, and the velocity of the moving cart was v_0. Express your answers in terms of "m" and "v_0".

5. What was the momentum of each cart after the collision? This would be the time after the carts had collided but before you stopped the target cart. Again, express your answers in terms of "m" and "v_0".

6. How did the sum of the two momentums for the carts before the collision compare to the sum of the two momentums for the carts after the collision? Were these two "total" momentums, before and after the collision, approximately the same or was one much bigger than the other?

7. Evaluate the following student statements. Identify ideas that are consistent with your ideas and others that are not consistent with your ideas.

Student A
"The moving cart lost all of its momentum during the collision."

Student B
"The target cart gained the momentum that the moving cart lost during the collision."

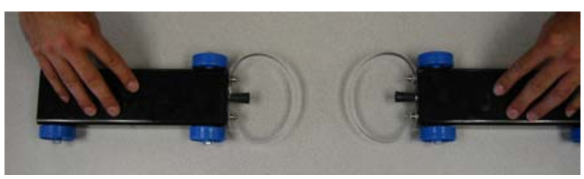
Check your work with your teacher

The Challenge

You will investigate how equal-mass objects behave when they're both moving before a one-dimensional head on collision.

Your Ideas about the Challenge

8. Predict what you think will happen to each cart if they are involved in a head-on collision in which they are both moving at the same speed before the collision.



The Investigation (continued)

a. Place the lab carts with their bumpers facing each other on the table top approximately 50 cm apart.
b. Using both hands, one student should push both carts towards one another so that they have the same approximate speed before they collide.
c. Take note of the magnitude and direction of the velocity of each cart after they collide.

9. How did the momentum of each cart compare before they collided? Did they have approximately the same momentum or did one cart have a lot more momentum than the other before the collision?

10. How should you show mathematically that the two carts were traveling in the opposite direction before the collision? Hint: is momentum a vector quantity? What do you do with the signs of oppositely directed vector quantities?

11. What would the resultant be if you added two vectors together that have the same magnitude but are in the opposite direction?

$\longrightarrow + \longleftarrow = ?$

12. What must have been an approximate value for the total momentum of this "system" <u>before</u> the two carts collided?

13. What happened to the direction of motion for <u>each</u> cart <u>after</u> the collision?

14. How did the momentum of each cart compare <u>after</u> they collided? Did they have approximately the same momentum or did one cart have a lot more momentum than the other cart <u>after</u> the collision?

15. What must have been an approximate value for the total momentum of this "system" <u>after</u> the two carts collided? Keep in mind the relative directions of their motion as well as the fact that momentum is a vector quantity.

16. How did the total momentum of the colliding cars <u>before</u> they collided compare to the total momentum <u>after</u> they collided? Was the total momentum unchanged <u>before</u> and <u>after</u> the collision or did the system appear to lose or gain momentum <u>after</u> the collision?

17. What appears to be the relationship for the total momentum of a system <u>before</u> and <u>after</u> there is an exchange of momentum?

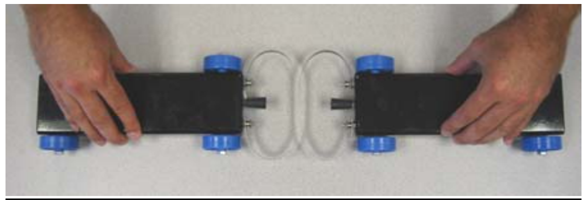
18. Evaluate the following student statement about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

"The total momentum for the two carts never changed. Since they both had the same speed and mass the total momentum before they collided was twice what it was for either cart. The same thing happened after they collided. The total momentum was mv + mv, or 2mv, before and after the collision."

Check your work with your teacher ☐

The Challenge

You will investigate how equal-mass objects behave when they both push off of each other in opposite directions.



Your Ideas about the Challenge

19. Predict what you think will happen to each cart if they are pushed together so that their bumpers are compressed and then they are released.

The Investigation (continued)

a. Place the two lab carts together so that their bumpers touch one another.
b. One student should push the lab carts into each other so that their bumpers are compressed somewhat.
c. This student should then attempt to release both carts at the same moment so that they are both released from rest at the same time.

20. What was the total momentum of this system before the two carts are released?

21. Did the two carts appear to have the same speed (and momentum) after they were released or was one cart moving a lot faster than the other one?

22. What must have been the total momentum after the two carts were released? Explain how you know.

23. Did the carts experience the same size force when they pushed off of each other or did one experience a greater force than the other? Explain how you know.

24. Explain how the directions compared for the forces that each cart experienced.

25. Evaluate the following student statements about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

<u>Student A</u>
"Each cart experienced the same size force and for the same period of time. They must have each received the same size impulse and the same size change in momentum."

<u>Student B</u>
"The only difference between the impulses and changes in momentum that they received was the direction. They got the same impulse but exactly in the opposite direction."

Check your work with your teacher ☐

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CONCEPT DEVELOPMENT

During the exploration activity you determined that the total momentum for an isolated system is constant. The idea that the total momentum of an isolated system is unchanged over time is known as **conservation of momentum**. A **system** is a group of objects that interact with each other. An **isolated system** would be a group of objects that <u>only</u> interact with each other and nothing else. If a system is truly isolated then there are no forces from outside of the system that would influence these objects during their interaction. If there are no outside (or external) forces acting on a system then momentum is conserved.

In this activity you will be working with some equations that can be used to make predictions about objects that are involved in what will be referred to as **elastic collisions**. An elastic collision is one in which the objects bounce off of one another after the collision.

Engagement Questions

A big truck runs into the rear of a small car that was at rest with its parking brake off and the transmission in neutral.

1. Will one vehicle experience a greater force during the collision or will they both experience the same size force? Explain how you know.

2. Will one vehicle experience the collision for a greater period of time or will they both experience the collision for the same length of time?

3. Will one vehicle experience a greater impulse during the collision or will they both experience the same size impulse? Explain how you know.

4. Will one vehicle experience a greater change in momentum during the collision or will they both experience the same size change in momentum? Explain how you know.

5. Will one vehicle experience a greater change in velocity during the collision or will they both experience the same size change in velocities? Explain how you know.

The Challenge

You will collect data and perform calculations for a collision between two objects that are of unequal mass.

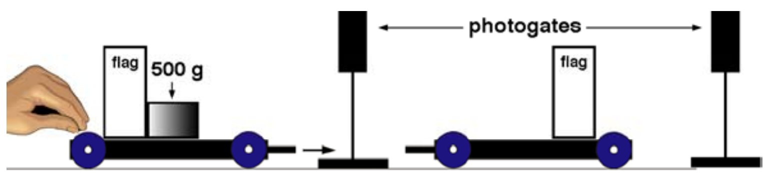
Your Ideas about the Challenge

6. What do you think would happen to the truck and the car mentioned previously if the truck ran into the car in a direct head-on collision while the car was initially at rest?

At each lab station you will find the following:
a computer with an interface, two photogates, an electronic balance, and two lab carts with flags affixed to the top. One cart will have an additional 500 g mass on top.

The Investigation

a. Find the mass of each cart using the electronic balance. Cart 1 is the cart that has the 500 g mass on top. Record their masses, expressed in kilograms, in the data table that follows.
b. The carts should be placed so that their bumpers face one another approximately 0.5 meters apart.
c. One of the photogates should be placed between the carts so that the flag on cart 1 will pass completely through the photogate before the carts collide.
d. The other photogate should be placed on the opposite side of the target cart so that the flag on that cart will pass through this gate, followed by the flag on the more massive cart, after the carts have collided together.
e. Start the data collection program.
f. Gently push cart 1 so that it passes through the first photogate and then strikes cart 2. Cart 2 should then pass through the second photogate followed by cart 1.
g. Record the velocities that you receive in the data table that follows.



Data

Cart 1 (kg)	Cart 2 (kg)

Cart 1 initial velocity (v_{01}, m/s)	Cart 2 final velocity (v_2, m/s)	Cart 1 final velocity (v_1, m/s)

7. Calculate the momentum for cart 1 before the collision. Be sure to label your answer with appropriate units.

$p_{01} = m_1 v_{0\,1} =$

8. What was the momentum for cart 2 before the collision? How do you know this?

9. What was the total momentum for your system before the collision? This would be the sum of the two momenta (plural for momentum) of the carts before the collision.

10. What was the momentum for cart 1 after the collision? Be sure to label your answer with appropriate units.

$p_1 = m_1 v_1 =$

11. Calculate the momentum for cart 2 after the collision. Be sure to label your answer with appropriate units.

$p_2 = m_2 v_2 =$

12. What was the total momentum for your system <u>after</u> the collision?

p_1 + p_2 =

13. How did the total momentum before the collision compare to the total momentum after the collision? Were they close in value or was one much bigger than the other?

Check your work with your teacher. ☐

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CONCEPT REFINEMENT

Review

When momentum is conserved, the sum of the momenta (plural for momentum) before a collision takes place is equal to the sum of the momenta after the collision takes place. This is true as long as there is nothing from outside of this system that exerts any forces on the objects that make up the system.

In the following set of exercises you will be using the idea of conservation of momentum in order to make predictions about the velocities and momenta for either of the two bodies involved in the problem. We will only consider two-body systems that are involved in one-dimensional (straight line) collisions in which the objects bounce off of one another.

An equation that you can use for the kind of problem described above uses the idea of conservation of momentum. The mathematical equation for conservation of momentum shows that the total momenta before the collision is equal to the total momenta after the collision.

total momentum before = total momentum after

$$m_1v_{0\,1} + m_2v_{0\,2} = m_1v_1 + m_2v_2$$

This equation involves an exchange of force and momenta between two objects m_1 and m_2. Their initial velocities have the double subscripts ($_{01}$ and $_{02}$) while their final velocities have a single subscript. In either case, this mathematical technique will help you to keep track of which velocity you are using and which object that velocity belongs to. It's possible that both objects could be moving both before and after the collision. This would require you to deal with 4 different velocities.

1. Indicate what each of the following four velocities stands for.

v_{01} :________________ v_{02} :__________________ v_1 :__________________ v_2 :_________________

In many exercises one of the objects might start from rest before the collision or one of the objects might come to rest after the collision. In this case either its initial velocity would be zero or its final velocity would be zero.

2. What is the momentum for an object that has zero velocity?

A 3 kg skateboard is rolling down the sidewalk at 4 m/s when it collides with a 1 kg skateboard that was initially at rest. If the 1 kg skateboard moves off at 6 m/s after the collision what is the new velocity for the 3 kg skateboard?

To solve this problem you would use the equation you see below.

$$m_1v_{0\,1} = m_1v_1 + m_2v_2$$

3. Explain how the original equation you see towards the top of this page reduces to the form you see above for this particular problem.

In order to solve for the final velocity of the 3 kg skateboard you would have to do some algebra so that you had the final velocity of the 3 kg skateboard (v_1) all by itself on the right side of the equation.

$$\frac{m_1 v_{01} - m_2 v_2}{m_1} = v_1$$

4. What two algebraic steps would you have to do in order to get the reduced equation in the form that you see above?

After doing the algebra you would then substitute the values you were given for the mass and velocities for the two skateboards provided in the original problem.

5. What is the final velocity for the 3 kg skateboard?

Check your work with your teacher.

6. A 2,000 kg boat moving at 3 m/s strikes an 8,000 kg boat that was floating at rest. If the 8,000 kg boat moves away at 1 m/s after the collision what is the final velocity for the 2,000 kg boat after the collision?

7. Why did the final velocity for the 2,000 kg boat turn out to be a negative quantity? What direction would the 2,000 kg boat be moving in after the collision?

Check your work with your teacher.

A 500 kg cannon fires a 10 kg cannon ball.

8. What is the total momentum of this system before the cannon is fired? How do you know?

9. How many objects have momentum after the cannon is fired? What are those objects?

10. Write the equation that describes this situation using the idea of conservation of momentum.

11. If the cannon ball moves at 100 m/s after it is fired, how fast does the cannon move after it is fired?

12. In what direction does the cannon move after it is fired? Explain how you know this.

Check your work with your teacher.